Increasing Weather Risk: Fact or Fiction?

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Outline

Motivation I

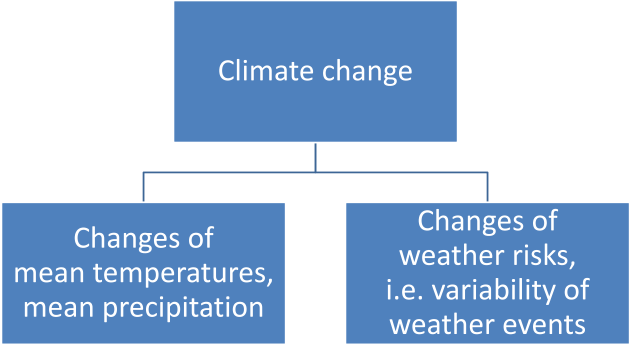

Figure: Climate change

Motivation II

- Undisputed statement among meteorologists and insurance companies: Weather risk / weather extremes increased due to climate change (e.g. IPCC (2007), Alexander et al.(2006), Munich Re (2011))
- Differentiation required with regard to:
    - weather event (temperature, rainfall, basic weather variables, weather indices)
    - region
    - time (continous change, jumps, unidirectional)

Motivation III

Standard procedure for testing a change of weather risk in meteorology

1. Definition of a weather index that measures an "‘extreme" (e.g. min / max temperature, length of dry spells, amount of rainfall within a certain period)
2. Determination of an observation period
3. Slope test (e.g. Mann-Kendall-Test) for the mean of the specific extreme weather index)

Alternatives

- Looking at quantiles of weather variables
- Local tests
- Change point tests
- Extreme value theory

Research questions and objectives

- Does weather risk increase for temperature related events as well as for precipitation related events?
- Is weather risk increasing everywhere?
- Does increasing weather risk of basic weather variables translate into increasing risk of (economically relevant) weather indices?
- Refinement of statistical testing procedure (local tests, change pool tests, quantile regression)

Outline

Tests

- t- test: the significance of the slope parameter
- change point test: whether there is a significant difference between the means in two sub intervals
- Mann Kendall test: a nonparametric test for trend or change point detection
- detect slope or change point

t test

$$m(t) = \alpha + \beta t \tag{1}$$

a significant slope parameter β?
The estimation of parameter:

$$\text{argmin}_{\alpha,\beta} \sum_t (T_t - \alpha - \beta t)^2 \tag{2}$$

The null hypothesis :$\beta = 0$, and the alternative: $\beta \neq 0$.

Change point test

Two windows L_1 (n_1) and L_2 (n_2) . Sample mean:

$$\mu_j = \sum_t^{n_j} T_t, j \in 1, 2, t \in L_j \tag{3}$$

The null hypothesis $\mu_1 = \mu_2$; the alternative $\mu_1 \neq \mu_2$. The test statistics:

$$V = \frac{n_1 n_2}{n_1 + n_2} \frac{(\hat{\mu}_1 - \hat{\mu}_2)^2}{\tilde{\sigma}^2} \xrightarrow{\mathcal{L}} \chi^2(1),$$

where $\tilde{\sigma}$ is from:

$$\tilde{r}_t = T_t - \hat{m}_h(t)$$

$$\hat{m}_h(t) = \frac{\sum_s K\{(s-t)/h\} T_s}{\sum_s K\{(s-t)/h\}}$$

Mann Kendall test

Test statistics:

$$S = \sum_{t=2}^{n} \sum_{s=1}^{t-1} \text{sign}(T_t - T_s)$$

For $n \to \infty$, S asymptotically normal:

$$S^* = \frac{S}{\sqrt{n(n-1)(2n+5)/18}} \xrightarrow{\mathcal{L}} \mathbf{N}(0,1).$$

Local Trend Tests

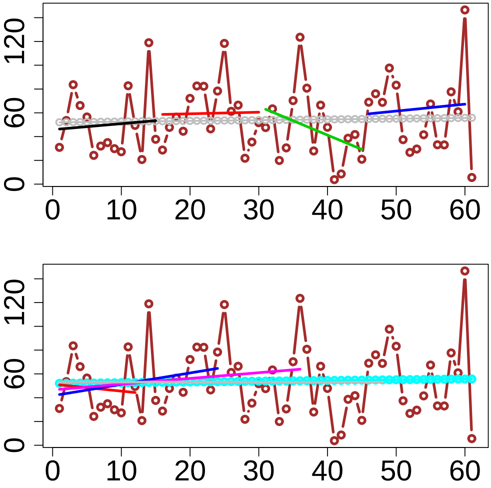

Figure: Demonstration

Local Trend Tests

- Apply test in a rolling window
- Consecutive significant signals
- τ (the minimum number of subsequent P-values that is eligible to create a summation measure)
- κ (the maximum number of insignificant P-values to drop everything to 0)

Quantile Regression

$\{t, Y_t\}_{t=1}^{T}$, $Y_t s$ independent.

$$Y_t = l(t) + \varepsilon_t, \tag{4}$$

where $\mathbb{P}(\varepsilon_t > 0) = \tau$.

$$\begin{aligned} \hat{l}(s) &= \operatorname{argmin}_\theta (1-\tau) \sum_{t=1} (\theta - Y_t) \mathbf{I}(\theta > Y_t) w_t \\ & \quad - \tau \sum_{t=1} (Y_t - \theta) \mathbf{I}(\theta \leq Y_t) w_t, \end{aligned}$$

where $w_t = K\{(t-s)/h\}$, and $K(.)$ a kernel function

Cumulative Indices

- Growing Degree Days (GDD) (Temperature)

$$GDD = \sum_{j=\tau BS}^{\tau ES} \max\{(T_{\max,j} + T_{\min,j})/2 - T_{\mathbf{base},j}, 0\},$$

  where BS and ES are the beginning and the end of a vegetation period.

- Cumulative Rainfall Index (CRI) (Rainfall):

$$CRI = \sum_{j=\tau BS}^{\tau ES} R_j. \tag{5}$$

Extreme Indices

- Frost Days Index (FDI)(Temperature):

$$FDI = \sum_{j=\tau B}^{\tau E} \mathbf{I}(T_j < T_{\min,j}),$$

where B and E are the beginning and the end of a cumulation period.

- Potential Flood Indicator (PFI) (Rainfall):

$$PFI = \max_{\tau \in \{1,\ldots,365-s+1\}} \left(\sum_{j=\tau}^{s+\tau-1} R_j \right), \tag{6}$$

which estimates the wettest s-day period in the year, e.g. five days.

Empirical application: Data

- Berlin Tempelhof: Jan, 1st, 1948- Mar, 22nd, 2011
- Taipei: Jan, 1st, 1910- Dec, 31st, 2008.
- Iowa: Jan, 1st, 1905- April, 6th, 2011

Illustration of Quantiles

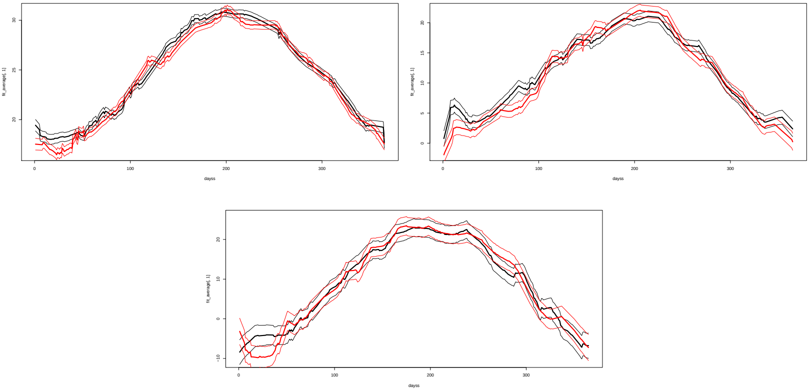

Figure: %90 quantile and the band the temperature from 1989 – 1998 averaged over a five year interval for city Taiwan, Berlin and Iowa, chronological order: black, red

Illustration of Quantiles

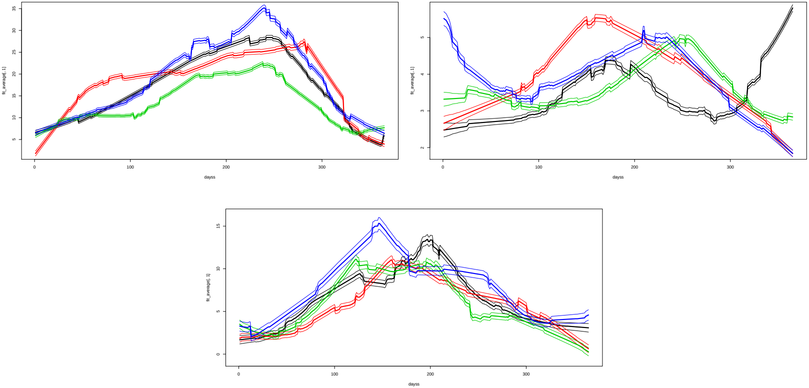

Figure: %90 quantile and the band the rainfall from 1989 – 2010 averaged over a five year interval for city Taiwan, Berlin and Iowa, chronological order: black, red, green, blue

Empirical application: Local test for FDI and PFI

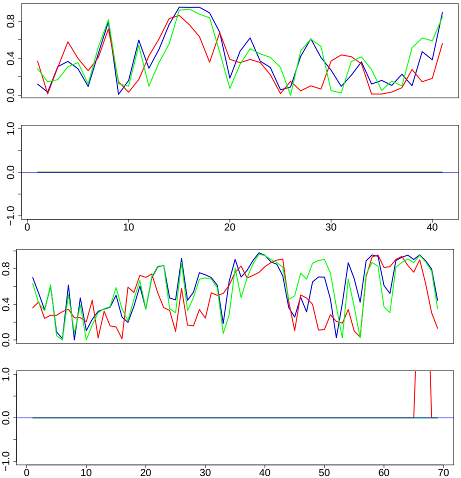

Figure: Local test results for temperatures four cities, FDI, *t* test (blue), change point test (red), Mann Kendall test (green)

Empirical application: Local test for FDI and PFI

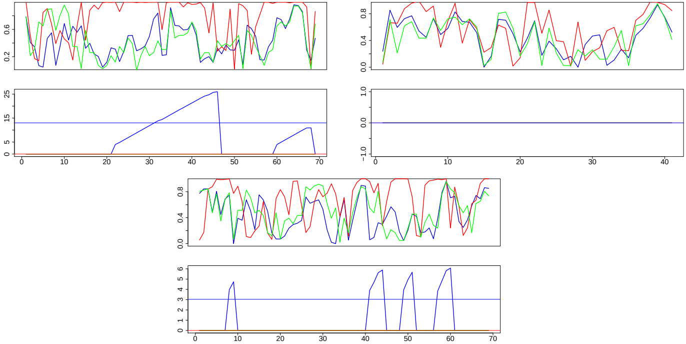

Figure: Local test results for rainfall four cities, PFI, t test (blue), change point test (red), Mann Kendall test (green), Taipei, Berlin, Iowa

Empirical application: Quantile for GDD

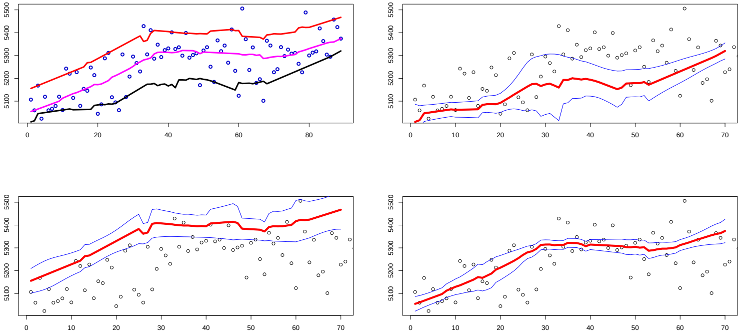

Figure: Plot quantile GDD Taiwan, left upper panel: %75, %50, %25 quantile curve, right upper panel: %25 quantile curve and its band, left lower panel: %75 quantile curve and its band, right lower panel: %50 quantile curve and its band

Empirical application: Quantile for CRI

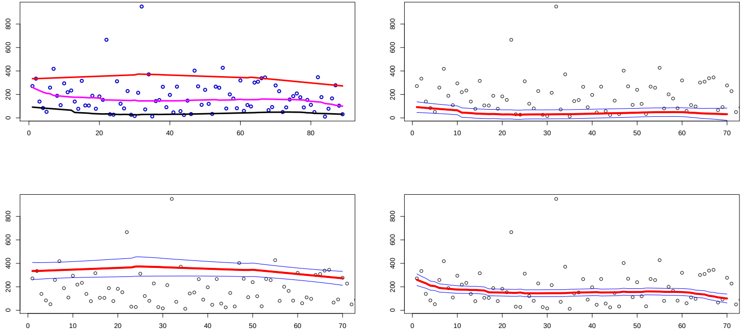

Figure: Plot quantile CRI Taiwan, left upper panel: %75, %50, %25 quantile curve, right upper panel: %25 quantile curve and its band, left lower panel: %75 quantile curve and its band, right lower panel: %50 quantile curve and its band

Conclusions

- There are spatial variabilities and time variabilities
- For GDD, normally the lower quantiles are more volatile.
- For CRI, Berlin: decreasing, Iowa: slightly increasing, Taipei: No change
- The quantile curves at different levels behave differently
- t-test, change point test have more power
- For FDI, there is no trend detected for both Berlin and Iowa.
- For PFI, Iowa and Taipei exhibit trends

Appendix

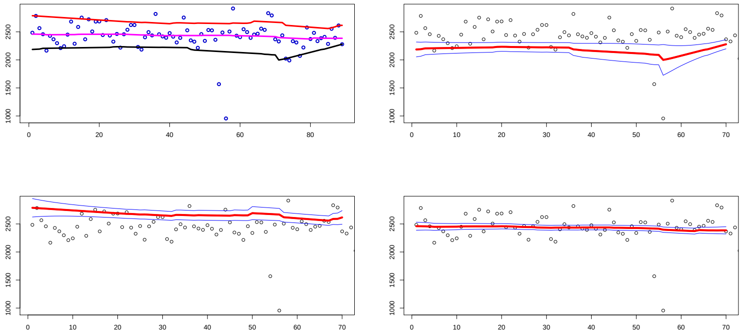

Figure: Plot quantile GDD Iowa (temperature)

Appendix

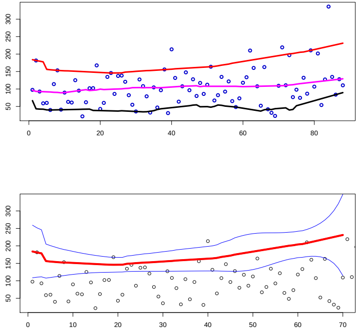

Figure: Plot quantile CRI May Iowa

Appendix

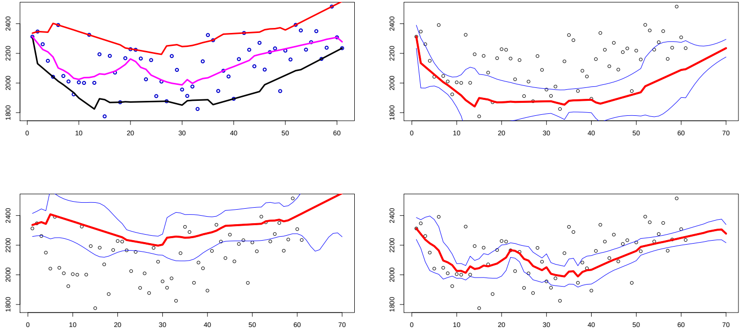

Figure: Plot quantile GDD Berlin

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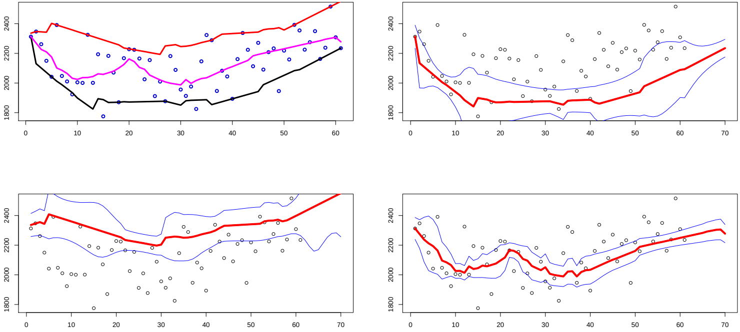

Figure: Plot quantile GDD Berlin

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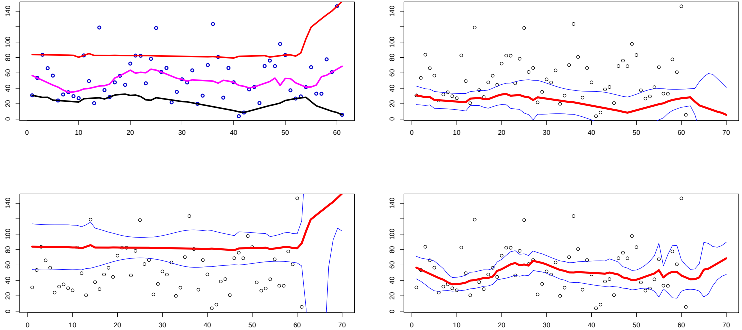

Figure: Plot quantile CRI Berlin (Rainfall)

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